

## Chapter

## 12

# Ratio and Rate

## Learning Objectives

After completing this chapter, you will be able to

- understand the meaning of ratio and rate.
- know the notations of  $a : b$  and  $a : b : c$ .
- apply ratio and rate to solve real-life problems including mensuration problems.



1



2

Happy Cow milk in three different packages are available in a supermarket. Which one is the most economical? Does the most economical one fit your need?



3



4



## Preview

[Basic knowledge required for this chapter.]

### Basic Knowledge

#### 1. Expanding fractions

Example: Rewrite  $\frac{2}{3}$  as a fraction with denominator 12.

$$\text{Solution: } \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

#### 2. Simplifying fractions

Example: Simplify  $\frac{9}{24}$ .

$$\text{Solution: } \frac{9}{24} = \frac{3 \times 3}{3 \times 8} = \frac{3}{8}$$

## 12.1 Ratio



### Class Activity 12.1

**Aim:** To investigate the concept of ratio from daily experience

After *adjusting* the size of photo A with a computer, photos B, C and D are obtained.



Photo A

adjust 調整





Photo B



Photo C



Photo D

- The image in photo B / C / D is not *distorted*.
- (a) As indicated in the figure on the right, measure the length and width for each photo above and complete the following table.

	Photo A	Photo B	Photo C	Photo D
$x$	8	7	6	6
$y$	6	4	4.5	8
$x \cdot y$	48	28	27	48
$\frac{x}{y}$	$\frac{8}{6} = \frac{4}{3}$	$\frac{7}{4}$	$\frac{6}{4.5} = \frac{4}{3}$	$\frac{6}{8} = \frac{3}{4}$



- From the above table, the values of  $x \cdot y$  of photo A and photo D are the same.
- From the above table, the values of  $\frac{x}{y}$  of photo A and photo C are the same.

3. From the results of 1 and 2, determine which expression in  $x$  and  $y$  can be used to check the distortion of the image after adjusting the photo size.

$$\frac{x}{y}$$

**Now I see ...**

To ensure the image of a photo is not distorted after adjusting its size, the value of  $\frac{x}{y}$  should be considered.



For two quantities  $x$  and  $y$  (where  $x \neq 0$  and  $y \neq 0$ ) of the same kind, the value  $\frac{x}{y}$  is called the **ratio** of  $x$  to  $y$ , and is usually written as  $x : y$ .

Since ratio is the quotient of two quantities of the same kind, it has no units. For example, Table 12.1 shows the ratio of the length and width ( $x : y$ ) for each photo in the Class Activity 12.1.

	Photo A	Photo B	Photo C	Photo D
Ratio of the length and width ( $x : y$ )	$8 : 6 = \frac{8}{6}$	$7 : 4 = \frac{7}{4}$	$6 : 4.5 = \frac{6}{4.5}$	$6 : 8 = \frac{6}{8}$

Table 12.1

**Notes:** (a) Ratios can be simplified.

For example,  $6 : 8 = \frac{6}{8} = \frac{3}{4} = 3 : 4$

(b)  $\therefore 8 : 6 = \frac{8}{6} = \frac{4}{3} = 4 : 3$

$6 : 4.5 = \frac{6}{4.5} = \frac{60}{45} = \frac{4}{3} = 4 : 3$

$\therefore$  We call  $8 : 6$  and  $6 : 4.5$  **equivalent ratios**.

- (c) In general,  $x : y$  and  $y : x$  are not equal. The images in photo A (where  $x : y = 8 : 6$ ) and photo D (where  $x : y = 6 : 8$ ) in the Class Activity 12.1 are not similar.

**Example 12.1** Simplification of ratios

Simplify the following ratios.

(a)  $98 : 84$

(b)  $0.32 : 0.8$

(c)  $\frac{3}{2} : \frac{3}{4}$

(d)  $0.5 \text{ kg} : 200 \text{ g}$

**Solution**

$$\begin{aligned} \text{(a)} \quad 98 : 84 &= \frac{98}{84} \\ &= \frac{7}{6} \\ &= \underline{\underline{7 : 6}} \end{aligned}$$

*Alternative method:*

$$\begin{aligned} 98 : 84 &= \frac{98}{14} : \frac{84}{14} \\ &= \underline{\underline{7 : 6}} \end{aligned}$$

◀ Both sides are divided by 14, where 14 is the H.C.F. of 84 and 98.

$$\begin{aligned} \text{(b)} \quad 0.32 : 0.8 &= \frac{0.32}{0.8} \\ &= \frac{32}{80} \\ &= \frac{2}{5} \\ &= \underline{\underline{2 : 5}} \end{aligned}$$

◀  $\frac{0.32}{0.8} = \frac{0.32 \times 100}{0.8 \times 100} = \frac{32}{80}$

*Alternative method:*

$$\begin{aligned} 0.32 : 0.8 &= 0.32 \times 100 : 0.8 \times 100 \\ &= 32 : 80 \\ &= \underline{\underline{2 : 5}} \end{aligned}$$

◀  $a : b = ak : bk$ , where  $k \neq 0$ .

$$\begin{aligned} \text{(c)} \quad \frac{3}{2} : \frac{3}{4} &= \frac{\frac{3}{2}}{\frac{3}{4}} \\ &= \frac{3}{2} \times \frac{4}{3} \\ &= 2 \\ &= \underline{\underline{2 : 1}} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 0.5 \text{ kg} &= 0.5 \times 1\,000 \text{ g} \quad \leftarrow 1 \text{ kg} = 1\,000 \text{ g} \\ &= 500 \text{ g} \\ \therefore 0.5 \text{ kg} : 200 \text{ g} &= 500 \text{ g} : 200 \text{ g} \\ &= \frac{500}{200} \\ &= \frac{5}{2} \\ &= \underline{\underline{5 : 2}} \end{aligned}$$

Unify the units of the two quantities before finding their ratios.



unify 統一

**Classwork 12.1**

Simplify the following ratios.

(a)  $8 : 18$

(b)  $0.75 : 0.45$

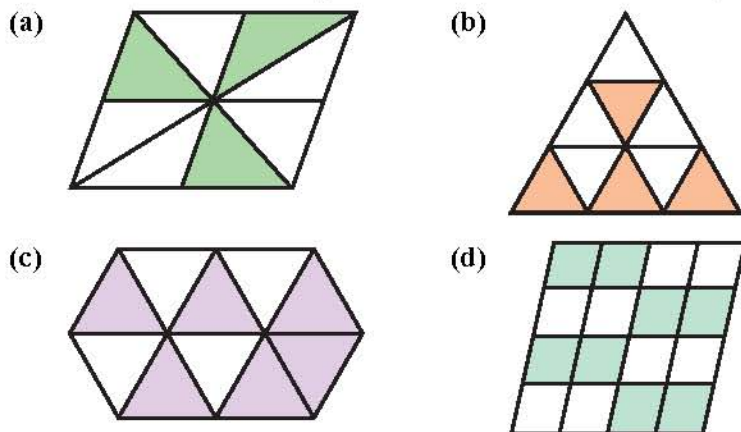
(c)  $6\frac{1}{2} : 1\frac{1}{4}$

(d)  $270 \text{ cm} : 0.9 \text{ m}$



## Example 12.2 Understanding ratios from figures

Each of the following is formed by identical figures. Find the ratio of the area of the shaded regions to that of the unshaded regions.



### Solution

- (a) Area of the shaded regions : Area of the unshaded regions =  $\underline{\underline{3 : 5}}$   
 (b) Area of the shaded regions : Area of the unshaded regions =  $\underline{\underline{4 : 5}}$   
 (c) Area of the shaded regions : Area of the unshaded regions =  $6 : 4$   
 $\quad\quad\quad = \underline{\underline{3 : 2}}$   
 (d) Area of the shaded regions : Area of the unshaded regions =  $8 : 8$   
 $\quad\quad\quad = \underline{\underline{1 : 1}}$

## Example 12.3 Finding unknowns with given ratios

- (a) If  $4 : 5 = x : 50$ , find the value of  $x$ .  
 (b) If  $2 : 3 = 16 : y$ , find the value of  $y$ .

### Solution

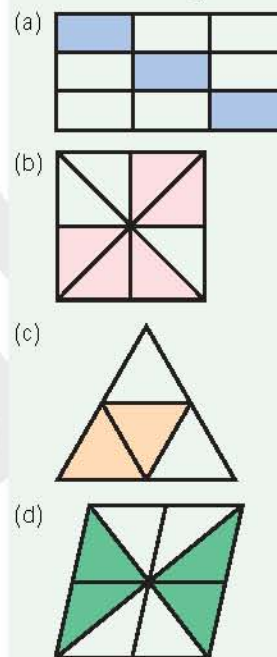
- (a)  $4 : 5 = x : 50$   
 $\frac{4}{5} = \frac{x}{50}$   
 $\frac{4}{5} \times 50 = \frac{x}{50} \times 50$   
 $x = \underline{\underline{40}}$

*Alternative method:*

$$\begin{aligned} \therefore 4 : 5 &= 4 \times 10 : 5 \times 10 \\ &= 40 : 50 \\ \therefore x &= \underline{\underline{40}} \end{aligned}$$

## Classwork 12.2

Each of the following is formed by identical figures. Find the ratio of the area of the shaded regions to that of the unshaded regions.



## Classwork 12.3

- (a) If  $8 : 3 = x : 9$ , find the value of  $x$ .  
 (b) If  $5 : 7 = 35 : y$ , find the value of  $y$ .

(b)  $2 : 3 = 16 : y$

$$\frac{2}{3} = \frac{16}{y}$$

$$2y = 3 \times 16$$

$$y = \frac{48}{2}$$

$$= \underline{\underline{24}}$$

*Alternative method:*

$$\therefore 2 : 3 = 2 \times 8 : 3 \times 8$$

$$= 16 : 24$$

$$\therefore y = \underline{\underline{24}}$$

### Example 12.4 Finding ratios with given equations

In the following,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $m$  and  $n$  are non-zero.

(a) If  $3a = 4b$ , find  $a : b$ .

(b) If  $\frac{c}{2} = \frac{d}{7}$ , find  $c : d$ .

(c) If  $m$  is 3 times of  $n$ , find  $m : n$ .

#### Solution

(a)  $3a = 4b$

$$\frac{3a}{b} = \frac{4b}{b}$$

$$\frac{3a}{b} \times \frac{1}{3} = \frac{4}{1} \times \frac{1}{3}$$

$$\frac{a}{b} = \frac{4}{3}$$

$$\therefore a : b = \underline{\underline{4 : 3}}$$

(b)  $\frac{c}{2} = \frac{d}{7}$

$$\frac{c}{2d} = \frac{1}{7}$$

$$\frac{c}{d} = \frac{2}{7}$$

$$\therefore c : d = \underline{\underline{2 : 7}}$$

(c)  $m = 3n$

$$\frac{m}{n} = \frac{3}{1}$$

$$\therefore m : n = \underline{\underline{3 : 1}}$$

#### Classwork 12.4

In the following,  $p$ ,  $q$ ,  $r$ ,  $s$ ,  $u$  and  $v$  are non-zero.

(a) If  $2p = 7q$ , find  $p : q$ .

(b) If  $\frac{r}{5} = \frac{s}{3}$ , find  $r : s$ .

(c) If  $u$  is  $\frac{3}{4}$  of  $v$ , find  $u : v$ .

### Example 12.5 Problems involving simplification of ratios

David made a model in 2 weeks. Carey made the same model in 16 days. Find the ratio of the number of days of David's work to that of Carey's work.

#### Solution

$$\begin{aligned} 2 \text{ weeks} &= 2 \times 7 \text{ days} \\ &= 14 \text{ days} \end{aligned}$$

$$\begin{aligned} \text{Number of days of David's work} : \text{Number of days of Carey's work} &= 14 : 16 \\ &= \underline{\underline{7 : 8}} \end{aligned}$$

### Example 12.6 Problems involving simplification of ratios

There are 40 students in S1C, and 4 of them are absent.

- Find the ratio of the number of presentees to the total number of students.
- Find the ratio of the number of absentees to the number of presentees.



#### Solution

$$\begin{aligned} \text{(a) Number of presentees} &= 40 - 4 \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{Number of presentees} : \text{Total number of students} &= 36 : 40 \\ &= \underline{\underline{9 : 10}} \end{aligned}$$

$$\begin{aligned} \text{(b) Number of absentees} : \text{Number of presentees} &= 4 : 36 \\ &= \underline{\underline{1 : 9}} \end{aligned}$$

### Example 12.7 Distributing objects in a ratio

A sum of \$1 500 is shared between A and B in the ratio of 3 : 2. How much does each of them get?

### Classwork 12.5

Rainbow is preparing a birthday party and she bought 2 dozens packs of lemon tea and 15 packs of green tea. Find the ratio of the number of packs of lemon tea to that of green tea.

### Classwork 12.6

There are 1 200 students in a school, and 640 of them are female.

- Find the ratio of the number of male students to the total number of students.
- Find the ratio of the number of female students to the number of male students.

### Classwork 12.7

Tea A and tea B are mixed in the ratio of 3 : 5 by weight. If the mixture is 24 kg, what is the weight of each kind of tea in the mixture?



**Solution**

When the sum is equally divided into  $3 + 2 = 5$  portions, A gets 3 portions and B gets 2 portions.

$$\begin{aligned}\therefore \text{A's share} &= \$1\,500 \times \frac{3}{5} \\ &= \underline{\underline{\$900}}\end{aligned}$$

$$\begin{aligned}\therefore \text{B's share} &= \$1\,500 \times \frac{2}{5} \\ &= \underline{\underline{\$600}}\end{aligned}$$

**Example 12.8** Finding total amount with given ratios

Henry spends \$13 875 out of his income and saves the rest monthly. It is known that the ratio of his savings to expenditure is 2 : 3.

- Find the monthly savings of Henry.
- Find the monthly income of Henry.

**Solution**

- (a) Let \$ $x$  be the monthly savings of Henry.

$$x : 13\,875 = 2 : 3$$

$$\frac{x}{13\,875} = \frac{2}{3}$$

$$\begin{aligned}x &= \frac{2}{3} \times 13\,875 \\ &= 9\,250\end{aligned}$$

$$\therefore \text{The monthly savings of Henry is } \underline{\underline{\$9\,250}}.$$

$$\begin{aligned}\text{(b) The monthly income of Henry} &= \$13\,875 + \$9\,250 \\ &= \underline{\underline{\$23\,125}}\end{aligned}$$

**Classwork 12.8**

It is known that the ratio of number of males and females in a city is 11 : 12. There are 3 300 000 males in the city.

- Find the number of females in the city.
- Find the total population of the city.

**Skills Upgrading Corner 12.1**

1. Simplify the following ratios.

(a)  $\frac{1}{4} : \frac{1}{12}$

(b)  $3 : \frac{3}{8}$

(c)  $0.3 : 0.04$

(d)  $2.4 : 0.03$

2. (a) If  $\frac{1}{3} : x = 2 : 3$ , find the value of  $x$ .

(b) If  $0.5 : 6 = y : 9$ , find the value of  $y$ .

3. The daily salaries of Mr. Wong and Mr. Lee are \$500 and \$400 respectively. If Mr. Wong and Mr. Lee have worked 15 days and 20 days this month respectively, find the ratio of Mr. Wong's salaries to Mr. Lee's this month.
4. Mario and Louis have 20 candies in total. If Mario is going to give  $\frac{1}{3}$  of his candies to Louis, the ratio of the number of candies of Mario to that of Louis will be 2 : 3.
- (a) How many candies does Louis have after getting some from Mario?
- (b) How many candies does Mario have before giving some to Louis?



## Exercise 12A

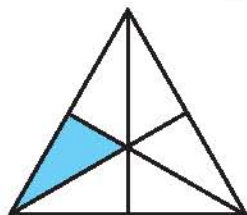
### Level 1

Simplify the following ratios. (1 – 4)

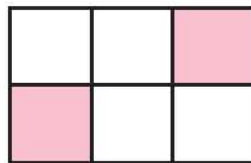
1. (a) 25 : 45 (b) 250 : 625 (c) 320 : 360
2. (a)  $\frac{5}{12} : \frac{2}{3}$  (b)  $\frac{7}{16} : \frac{7}{4}$  (c)  $\frac{7}{5} : \frac{14}{3}$
3. (a) 0.75 : 0.25 (b) 1.5 : 0.35 (c) 1.6 : 0.48
4. (a) 3 weeks : 35 days (b) \$6 : 75¢ (c) 120 cm : 1.8 m

5. Each of the following is formed by identical figures. Find the ratio of the area of the shaded regions to that of the unshaded regions.

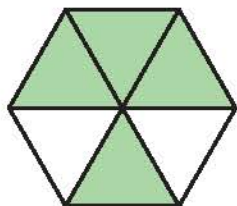
(a)



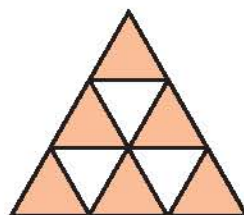
(b)



(c)



(d)



6. (a) If  $8:5 = x:55$ , find the value of  $x$ .  
 (b) If  $4:3 = 24:y$ , find the value of  $y$ .
7. The annual average rainfall of Beijing and Hong Kong is 580 mm and 2 200 mm respectively. Find the ratio of the annual average rainfall of Beijing to that of Hong Kong.
8. John is 140 cm tall. His father is 1.8 m tall. Find the ratio of John's height to his father's.
9. The marked price of a watch is \$480. The selling price is \$400.  
 (a) Find the discount.  
 (b) Find the ratio of the discount to the marked price.
10. There are 56 teachers in Ernest Secondary School. Among them, 24 are male teachers.  
 (a) Find the ratio of the number of female teachers to the total number of teachers.  
 (b) Find the ratio of the number of male teachers to that of female teachers.
11. Paul earns \$8 400 a month. The ratio of his spending to his savings monthly is 5:2. Find his monthly savings.
12. In 900 mL of juice mixture, the ratio of the volume of apple juice to that of pear juice is 2:1. Find the respective volumes of the juices in mL.
13. The area of a house is  $135 \text{ m}^2$ . If the area of the house to that of the garden is 3:8, find the area of the garden.
14. A sum of money is divided into two portions in the ratio of 5:7. If the smaller portion is \$1 300, find the amount of the sum.



### Level 2

15. Simplify the following ratios.
- (a)  $0.625 : \frac{3}{8}$                       (b)  $1.25 : 3\frac{1}{3}$                       (c)  $\frac{2}{9} : 1.5$   
 (d)  $40 \text{ minutes} : 2\frac{1}{2} \text{ hours}$                       (e)  $25 \text{ cm}^2 : 10 \text{ mm}^2$                       (f)  $0.000 32 \text{ m}^3 : \frac{8}{5} \text{ cm}^3$
16. (a) If  $x:(x+3)=5:6$ , find the value of  $x$ .  
 (b) If  $y:5=(y+4):7$ , find the value of  $y$ .



17. (a) Simplify  $10 : 16$ .  
 (b) Simplify  $(10 + 2) : (16 + 2)$ .  
 (c) If  $k \neq 0$ , is the expression ' $a : b = (a + k) : (b + k)$ ' correct?
18. In the following,  $x$ ,  $y$ ,  $u$ ,  $v$ ,  $s$  and  $t$  are non-zero.  
 (a) If  $2x = 5y$ , find  $x : y$ .  
 (b) If  $\frac{u}{4} = \frac{v}{9}$ , find  $u : v$ .  
 (c) If  $s$  is 2.5 times of  $t$ , find  $s : t$ .
19. For a kind of pills, the amount taken is suggested as follows:  
 Adult: 4 times a day, 4 pills each time  
 Child: 3 times a day, 2 pills each time  
 (a) Find the total number of pills suggested to be taken by an adult each day.  
 (b) Find the total number of pills suggested to be taken by a child each day.  
 (c) Find the ratio of the total number of pills suggested to be taken by an adult to that by a child each day.
20. There are 42 \$10 coins and some \$2 coins in a bag. It is given that the ratio of the number of \$10 coins to that of \$2 coins is 3 : 7.  
 (a) How many \$2 coins are there in the bag?  
 (b) What is the total value of all the coins in the bag?
21. There are 36 students in S1B. Some of them bring their own lunch and the others buy lunch boxes. If  $\frac{1}{5}$  of the students who usually bring their own lunch buy lunch boxes today, the ratio of students bringing their own lunch to those buying lunch boxes will be 1 : 2. How many students are there who usually bring their own lunch?
22. The ratio of the capacities of a large tank to that of a small tank is 7 : 2. If the capacity of the large tank is 5 L more than that of the small tank, find the capacity of the small tank.
23. In city A with a population of 5 000 000, the ratio of the number of mobile phone users to that of non-mobile phone users is 3 : 1. In city B with a population of 7 500 000, the ratio of number of mobile phone users to that of non-mobile phone users is 9 : 1. Which city has more non-mobile phone users? Explain briefly.



24. Nicholas goes to school and returns home by bus or MTR. It is given that the total fare for 3 bus trips is equal to 5 MTR trips.
- Find the ratio of the bus fare to MTR fare for each trip.
  - Nicholas travelled by bus 4 times and by MTR 6 times to and from school last week. If the total bus fare was \$38, find the MTR fare for each trip.



25. There is 40 mL of alcohol in a 1 000 mL mixture of alcohol and water.
- Find the ratio of the volume of alcohol to that of water in the mixture.
  - If 200 mL of alcohol is added to the mixture, how many times is the concentration of alcohol in the new mixture equal to that in the original mixture?

[Hint: Concentration of alcohol =  $\frac{\text{Volume of alcohol}}{\text{Volume of mixture}} \times 100\%$ ]

## 12.2 Ratios of Three Like Quantities

Ratio can also be used to express the relation among 3 like quantities.

For example: If  $a : b = 1 : 2$  and  $b : c = 2 : 3$ ,

then  $a : b : c = 1 : 2 : 3$ .

Conversely,

if  $x : y : z = 3 : 4 : 5$ ,

then  $x : y = 3 : 4$  and  $y : z = 4 : 5$ .

$a : b = \frac{a}{b}$ , but  $a : b : c$  cannot be expressed as a fraction.



### Example 12.9 Simplification of ratios

Simplify the following ratios.

- $4 : 6 : 10$
- $0.5 : 2 : 3.5$

#### Solution

$$\begin{aligned} \text{(a)} \quad 4 : 6 : 10 &= \frac{4}{2} : \frac{6}{2} : \frac{10}{2} \\ &= \underline{2 : 3 : 5} \end{aligned}$$

◀ All quantities are divided by 2, where 2 is the H.C.F. of 4, 6 and 10.

$$\begin{aligned} \text{(b)} \quad 0.5 : 2 : 3.5 &= 0.5 \times 10 : 2 \times 10 : 3.5 \times 10 \\ &= 5 : 20 : 35 \\ &= \frac{5}{5} : \frac{20}{5} : \frac{35}{5} \\ &= \underline{1 : 4 : 7} \end{aligned}$$

◀ All quantities are divided by 5, where 5 is the H.C.F. of 5, 20 and 35.

### Classwork 12.9

Simplify the following ratios.

- $15 : 6 : 9$
- $0.8 : 2.8 : 1.2$

### Example 12.10 Problems involving simplification of ratios

HSBC Main Building (H), Central Plaza (C) and Two International Finance Centre (I) are commercial buildings in Hong Kong. Their respective heights are around 180 m, 380 m and 420 m.



- (a) Find the ratio of the heights of H, C and I.  
 (b) Find the ratio of the heights of H and I.

#### Solution

(a) Ratio of heights of H, C and I = 180 m : 380 m : 420 m

$$= \frac{180 \text{ m}}{20 \text{ m}} : \frac{380 \text{ m}}{20 \text{ m}} : \frac{420 \text{ m}}{20 \text{ m}}$$

$$= \underline{\underline{9 : 19 : 21}}$$

(b) Ratio of heights of H and I = 9 : 21

$$= \frac{9}{3} : \frac{21}{3}$$

$$= \underline{\underline{3 : 7}}$$

### Example 12.11 Finding ratios of 3 quantities with given ratios of 2 quantities

- (a) Given  $a : b = 1 : 4$  and  $b : c = 4 : 5$ , find  $a : b : c$ .  
 (b) Given  $a : b = 5 : 3$  and  $b : c = 6 : 9$ , find  $a : b : c$ .

#### Solution

(a)

$$\begin{array}{lcl} a : b & = & 1 : 4 \\ b : c & = & 4 : 5 \end{array}$$


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$$\therefore a : b : c = \underline{\underline{1 : 4 : 5}}$$

(b)

$$\begin{array}{lcl} a : b & = & 5 : 3 \\ b : c & = & 6 : 9 \end{array}$$


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$$\therefore \begin{array}{lcl} a : b & = & 5 \times 2 : 3 \times 2 \\ b : c & = & 6 : 9 \end{array}$$


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$$\therefore a : b : c = \underline{\underline{10 : 6 : 9}}$$

### Classwork 12.10

Up till the end of 2005, the average daily traffic of Cross-Harbour Tunnel (C), Eastern Harbour Crossing (E) and Western Harbour Crossing (W) are around 120 000 vehicles, 64 000 vehicles and 40 000 vehicles respectively.

- (a) Find the ratio of the traffic of C, E and W.  
 (b) Find the ratio of the traffic of C to E, and that of C to W.

### Classwork 12.11

- (a) Given  $x : y = 4 : 7$  and  $y : z = 7 : 9$ , find  $x : y : z$ .  
 (b) Given  $x : y = 6 : 5$  and  $y : z = 10 : 7$ , find  $x : y : z$ .



**Example 12.12**

Finding ratios of 3 quantities with given ratios of 2 quantities

- (a) Given  $a : b = 2 : 3$  and  $b : c = 4 : 1$ , find  $a : b : c$ .  
 (b) Given  $a : b = 2 : 7$  and  $a : c = 5 : 3$ , find  $a : b : c$ .

**Solution**

$$\begin{array}{rcl} \text{(a)} & a : b & = 2 : 3 \\ & b : c & = 4 : 1 \end{array}$$

$$\therefore \begin{array}{rcl} a : b & = 2 \times 4 : 3 \times 4 \\ b : c & = 4 \times 3 : 1 \times 3 \end{array}$$

$$\therefore a : b : c = 8 : 12 : 3$$

$$\begin{array}{rcl} \text{(b)} & a : b & = 2 : 7 \\ & a : c & = 5 : 3 \end{array}$$

$$\therefore \begin{array}{rcl} a : b & = 2 \times 5 : 7 \times 5 \\ a : c & = 5 \times 2 : 3 \times 2 \end{array}$$

$$\therefore a : b : c = 10 : 35 : 6$$

**Example 12.13**

Finding ratios of 3 quantities with given equations

In the following,  $a$ ,  $b$  and  $c$  are non-zero.

- (a) If  $\frac{a}{b} = 4$  and  $b = 3c$ , find  $a : b : c$ .  
 (b) If  $3a = b = 4c$ , find  $a : b : c$ .

**Solution**

- (a) [Analysis: Find  $a : b$  and  $b : c$  first, then find  $a : b : c$  from  $a : b$  and  $b : c$ .]

$$\therefore \frac{a}{b} = 4$$

$$\therefore a : b = 4 : 1$$

$$\therefore b = 3c$$

$$\frac{b}{c} = 3$$

$$\therefore b : c = 3 : 1$$

$$\therefore a : b = 4 : 1 = 4 \times 3 : 1 \times 3 = 12 : 3$$

$$b : c = 3 : 1 = 3 : 1 = 3 : 1$$

$$\therefore a : b : c = 12 : 3 : 1$$

**Classwork 12.12**

- (a) Given  $x : y = 3 : 5$  and  $y : z = 2 : 1$ , find  $x : y : z$ .  
 (b) Given  $x : y = 7 : 4$  and  $x : z = 3 : 2$ , find  $x : y : z$ .

**Classwork 12.13**

In the following,  $a$ ,  $b$  and  $c$  are non-zero.

- (a) If  $a = 2b$  and  $b = 3c$ , find  $a : b : c$ .  
 (b) If  $a = 2b = 5c$ , find  $a : b : c$ .

$$\begin{aligned}
 \text{(b)} \quad & \because 3a = b = 4c \\
 & \therefore 3a = b \text{ and } b = 4c \\
 & \because 3a = b \\
 & \quad \frac{a}{b} = \frac{1}{3} \\
 & \therefore a : b = 1 : 3 \\
 & \because b = 4c \\
 & \quad \frac{b}{c} = 4 \\
 & \therefore b : c = 4 : 1 \\
 & \because a : b = 1 : 3 = 1 \times 4 : 3 \times 4 = 4 : 12 \\
 & \quad b : c = 4 : 1 = 4 \times 3 : 1 \times 3 = 12 : 3 \\
 & \therefore a : b : c = \underline{4 : 12 : 3}
 \end{aligned}$$

**Example 12.14** Application of skills in finding ratios of 3 quantities with given ratios of 2 quantities

The ratio of A's money to B's is 1 : 2, while the ratio of B's money to C's is 3 : 2. If they have a total of \$52, how much does each of them have?

**Solution**

$$\begin{aligned}
 \text{A's money : B's money} &= 1 : 2 \\
 &= 1 \times 3 : 2 \times 3 \\
 &= 3 : 6
 \end{aligned}$$

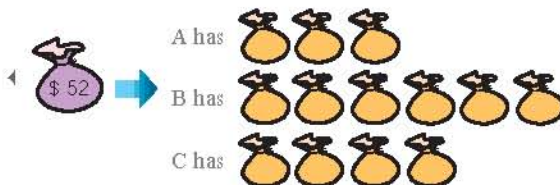
$$\begin{aligned}
 \text{B's money : C's money} &= 3 : 2 \\
 &= 3 \times 2 : 2 \times 2 \\
 &= 6 : 4
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{A's money : B's money : C's money} \\
 &= 3 : 6 : 4
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{A's money} &= \$52 \times \frac{3}{3+6+4} \\
 &= \$52 \times \frac{3}{13} \\
 &= \underline{\underline{\$12}}
 \end{aligned}$$

$$\begin{aligned}
 \text{B's money} &= \$52 \times \frac{6}{13} \\
 &= \underline{\underline{\$24}}
 \end{aligned}$$

$$\begin{aligned}
 \text{C's money} &= \$52 \times \frac{4}{13} \\
 &= \underline{\underline{\$16}}
 \end{aligned}$$



**Classwork 12.14**

The weight of an alloy made of metals X, Y and Z is 85 g. The ratio of the weights of metal X to metal Y is 4 : 1, while the ratio of the weights of metal Y to metal Z is 3 : 2. Find the weight of each metal in the alloy.



### Example 12.15 Finding total amount with given ratios

Amy, Betty and Candy are going to share the roses in the ratio of 1 : 2 : 3. If Candy gets 120 roses, find the total number of roses ordered.



### Solution

When the roses are divided into  $1 + 2 + 3 = 6$  portions, Amy gets 1 portion, Betty gets 2 portions and Candy gets 3 portions.

$$\begin{aligned}\therefore \text{The total number of roses} &= 120 \div \frac{3}{6} \\ &= 120 \times \frac{6}{3} \\ &= \underline{\underline{240}}\end{aligned}$$




### Classwork 12.15

A piece of cloth is cut into 3 portions in the ratio of 2 : 3 : 5. If the longest portion is 9 m long, how long is the piece of cloth originally?

### Skills Upgrading Corner 12.2

- (a) Given  $a : c = 1 : 3$  and  $b : c = 2 : 5$ , find  $a : b : c$ .

(b) Given  $3a = 2c$ ,  $b$  is twice of  $a$ , where  $a$ ,  $b$  and  $c$  are non-zero, find  $a : b : c$ .
- Sammy got 220 marks in three Mathematics tests. The ratio of his score in the first test to that in the second test is 8 : 9, while the ratio of his score in the second test to that in the third test is 6 : 7. Find Sammy's score in each test.
- The lengths of 3 wires are in the ratio of 13 : 15 : 12. If the shortest wire is 180 cm long, find the total length of the 3 wires.
- (a) The weights of Peggy and Queenie are 30 kg and 36 kg respectively. The ratio of the weight of Peggy to that of Rachel is 6 : 7. Arrange them in ascending order of their weights.

(b)  The ratio of the weight of Lily to that of Maggie is 6 : 7, and the weight of Nancy is 36 kg. Can you arrange them in ascending order of their weights? Explain briefly.







## Exercise 12B

### Level 1

1. Simplify the following ratios.

(a)  $4:8:16$

(b)  $18:24:40$

(c)  $60:30:48$

Find  $a : b : c$  for each of the following. (2 – 5)

2. (a)  $a:b=5:6$ ,  $b:c=1:2$

(b)  $a:b=3:7$ ,  $b:c=14:15$

(c)  $a:b=3:4$ ,  $b:c=5:1$

3. (a)  $a:b=4:3$ ,  $a:c=5:3$

(b)  $a:c=8:5$ ,  $b:c=3:2$

(c)  $b:c=5:3$ ,  $a:c=1:6$

4. (a)  $a:b=2:3$ ,  $c:b=3:2$

(b)  $b:a=7:4$ ,  $b:c=2:5$

(c)  $a:b=8:3$ ,  $c:b=3:10$

5. It is given that  $a$ ,  $b$  and  $c$  are non-zero.

(a)  $\frac{a}{b}=2$ ,  $\frac{b}{c}=5$

(b)  $\frac{a}{b}=3$ ,  $c=2b$

(c)  $3a=6b=5c$

6. The amounts of pocket money of Christine, Patrick and Paul are \$240, \$300 and \$180 respectively. Find the ratio of the pocket money of Christine, Patrick and Paul.

7. The heights of Tim, Calvin and Bruto are 155 cm, 170 cm and 180 cm respectively. Find the ratio of the heights of Tim, Calvin and Bruto.

8. The ages of Henry, his wife and his daughter are 32, 28 and 4 respectively.

(a) Find the ratio of ages of Henry, his wife and his daughter.

(b) Find the ratio of ages of Henry, his wife and his daughter after 4 years.

9. A wire of 36 m long is cut into three pieces. The ratio of their lengths is 1 : 2 : 3. Find the length of the middle piece.

10. A piece of ham which weighs 6 kg is cut into 3 pieces in the ratio of 3 : 5 : 4. Find the weight of the heaviest piece.

11. The ratio of the number of red, yellow and blue marbles in a bag is 2 : 1 : 6. If there are 18 red marbles, find the number of blue marbles in the bag.



12. The ratio of the weights of Alan, Bobo and Ivan is 3 : 2 : 4. If the weight of Alan is 54 kg, find the weight of Bobo.

**Level 2**

Simplify the following ratios. (13 – 14)

13. (a)  $0.56 : 1.6 : 4.2$  (b)  $\frac{1}{2} : \frac{1}{4} : \frac{3}{8}$

14. (a)  $\frac{2a}{3} : 5a : \frac{3a}{4}$  (b)  $\frac{a}{2} : \frac{a}{5} : \frac{a}{6}$

15. A piece of land with the area of  $1320 \text{ m}^2$  is divided among Peter, Chapman and Betty. The ratio of Peter's share to Chapman's is 2 : 3 and the ratio of Chapman's share to Betty's is 1 : 2. Find the area of Chapman's share.



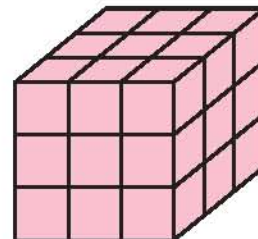
16. Vincent, Cindy and Jacky have *invested* \$25 000, \$50 000 and \$75 000 respectively in a business. The total profit made is \$54 000 and they share the profit according to the ratio of their investments.

- (a) Find the ratio of their investments.  
(b) How much profit does each of them get?

17. A bottle of juice is shared by Kenneth, Judy and Clara in the ratio of 2 : 3 : 5. If Clara gets 750 mL of juice, find the volume of juice in the bottle.

18. The ratio of the weights of 3 documents is 4 : 3 : 6. If the heaviest one weighs 390 g, find the total weight of all 3 documents.

19. The large cube in the figure is made up of 27 small cubes. All faces of this large cube are painted red. Find the ratio of the number of small cubes having one red face, two red faces and three red faces.



20. The lengths of the Tsing Ma Bridge (A), the Kap Shui Mun Bridge (B) and the Ting Kau Bridge (C) are around 2.2 km, 0.8 km and 1.2 km respectively.

- Find the ratio of the lengths of A, B and C.
- Mr. Chan drives through these three bridges at the same speed. If he drives through the Ting Kau Bridge in 72 seconds, how long does he take in total to drive through the other two bridges?



21. A sum of money is shared among Alfred, Bosco and Terence. Alfred's share is  $\frac{1}{2}$  of Bosco's and  $\frac{1}{3}$  of Terence's.

- Let \$ $x$  be Alfred's share. Express Bosco's and Terence's share in terms of  $x$ .
- Find the ratio of Alfred's, Bosco's and Terence's share.
- If the sum of money is \$7 200, find Alfred's share.

22. According to Mr. Wong's *will*,  $\frac{1}{4}$  of his *wealth* will be given to his son,  $\frac{1}{3}$  to his daughter, and the rest to Mrs. Wong.

- Let \$ $A$  be Mr. Wong's total wealth.
  - Express the respective share of his wealth by his son, his daughter and Mrs. Wong in terms of  $A$ .
  - Find the ratio of the share of Mr. Wong's wealth by his son, his daughter and Mrs. Wong.
- If Mrs. Wong's share of Mr. Wong's wealth is \$240 000, how much is their son's share of the wealth?

23. Since the ratio  $a : b$  can be written as  $\frac{a}{b}$ , Joseph suggests that the ratio  $a : b : c$  should be written as  $\frac{a}{\frac{b}{c}}$  or  $\frac{a}{\frac{b}{c}}$ . Determine whether his suggestion is correct. Explain briefly.

## 12.3 Rate

### Class Activity 12.2

**Aim:** To explore the concept of rate from daily experience

1. Tom walked 8 km in 2 hours.

Sam walked 15 km in 3 hours.

At a glance, it is difficult to say who walked faster. When the distance walked by each of them in an hour was considered, Tom walked 4 km, and Sam walked 5 km.

$\therefore$  Sam walked faster.



will 遺囑

wealth 財產



2. Susan wants to buy some apples. She finds that the apples in stall A and stall B are both of good quality, but their prices are different.
- Stall A sells 5 apples for \$12.
- Stall B sells 4 apples for \$10.
- At a glance, it is difficult to say apples from which stall are cheaper. When the average selling price is considered, apples from stall A are \$ 2.4 each, and apples from stall B are \$ 2.5 each.
- ∴ Apples from stall A are cheaper.



*Now I see ...*

In some situations, considering and comparing the quotients of two quantities of different kinds can help in solving the problem.



**Rate** is a quotient of two quantities.

e.g. In the above Class Activity,

1. distance divided by time is a speed.

$$\begin{aligned} \therefore \text{Speed of Tom} &= \frac{8 \text{ km}}{2 \text{ h}} \\ &= 4 \text{ km/h} \end{aligned}$$

◀ It means 4 km per hour.

2. the quotient of the price of apples divided by the number of apples is also a rate.

$$\begin{aligned} \therefore \text{The selling price of apples from stall A} &= \frac{\$12}{5 \text{ apples}} \\ &= \$2.4/\text{apple} \end{aligned}$$

◀ It means \$2.4 for each apple.

**Notes:** (a) Rates, in general, involve units, but ratios do not. For example, 5 km/h. (i.e. 5 kilometers per hour)

(b) In daily life, speed often refers to a rate with time as the denominator.

For example,

$$\text{driving speed} = \frac{\text{driving distance}}{\text{time}}$$

$$\text{typing speed} = \frac{\text{number of words}}{\text{time}}$$



The speed limit of cars on the road is 5 km/h.

### Example 12.16 Finding rates

Shirley can type 960 words in 20 minutes.  
Find

- her typing speed in words/min.
- her typing speed in words/h.

#### Solution

$$\begin{aligned} \text{(a) Typing speed} &= \frac{960 \text{ words}}{20 \text{ min}} \\ &= \underline{\underline{48 \text{ words/min}}} \end{aligned}$$

$$\begin{aligned} \text{(b) Number of words typed in an hour} &= 48 \times 60 \text{ words} \\ &= 2\,880 \text{ words} \end{aligned}$$

$$\therefore \text{Typing speed} = \underline{\underline{2\,880 \text{ words/h}}}$$

#### Alternative method:

$$\begin{aligned} \therefore 20 \text{ min} &= \frac{20}{60} \text{ h} \\ &= \frac{1}{3} \text{ h} \end{aligned}$$

$$\begin{aligned} \therefore \text{Typing speed} &= \frac{960 \text{ words}}{\frac{1}{3} \text{ h}} \\ &= 960 \times 3 \text{ words/h} \\ &= \underline{\underline{2\,880 \text{ words/h}}} \end{aligned}$$



### Classwork 12.16

28 eggs are sold for \$14. Find the price

- in \$ per egg.
- in \$ per dozen of eggs.

### Example 12.17 Finding unknowns with given rates

Mrs. Cheung has bought 2 kg of lychees for \$18.

- Find the selling price in \$/kg.
- Find the selling price of 7 kg of lychees.

#### Solution

$$\begin{aligned} \text{(a) Selling price} &= \frac{\$18}{2 \text{ kg}} \\ &= \underline{\underline{\$9/\text{kg}}} \end{aligned}$$

$$\begin{aligned} \text{(b) Selling price of 7 kg of lychees} &= \$9 \times 7 \\ &= \underline{\underline{\$63}} \end{aligned}$$

Consider the operation of units:  $\text{kg} \times \frac{\$}{\text{kg}} = \$$



### Classwork 12.17

The cost of petrol for a lorry travelling 140 km is \$308.

- Find the cost of petrol for the lorry in \$/km.
- Find the cost of petrol for the lorry travelling 28 km.



### Example 12.18 Using rates to perform simple analysis

Wendy *distributes* free newspapers to people passing through a street every day. She needs to distribute 2 200 newspapers today, and she has distributed 150 newspapers in the first 4 minutes.



- (a) Find her speed of distributing newspapers in newspapers/min.
- (b) If she keeps distributing newspapers at the same speed, can she complete distributing all newspapers in one hour? Explain briefly.

### Solution

$$\begin{aligned} \text{(a) Speed of distributing newspapers} &= \frac{150 \text{ newspapers}}{4 \text{ min}} \\ &= \underline{\underline{37.5 \text{ newspapers/min}}} \end{aligned}$$

$$\begin{aligned} \text{(b) Number of remaining newspapers} &= 2\,200 - 150 \\ &= 2\,050 \end{aligned}$$

$$\therefore \text{Time required} = \frac{2\,050}{37.5} \text{ min}$$

$$= 54\frac{2}{3} \text{ min}$$

$$< 60 \text{ min}$$

$$\therefore \underline{\underline{\text{She can complete distributing all newspapers in one hour.}}}$$



### Classwork 12.18

The *balance* of a mobile SIM card is \$40.8. It is known that the calling fee is \$6 for 5 minutes.

- (a) Find the fee rate in \$/min.
- (b) How long can one talk with this SIM card for?



### Skills Upgrading Corner 12.3

1. A newly designed car uses only 27 L of petrol to travel 648 km.
  - (a) Find the petrol consumption in km/L.
  - (b) If there is only 15 L of petrol in this car, how far can it travel?
  - (c) How much petrol is used for travelling 60 km?



distribute 分發

balance 餘額



2. If 40 British pounds (GBP) can be exchanged for 620 Hong Kong dollars (HKD),
- find the exchange rate from GBP to HKD. (Give the answer in HK\$/£.)
  - how much HKD can be exchanged for £100?
  - how much GBP can be exchanged for HK\$744?

⚡ £ is the symbol for British pound,  
e.g. HK\$10/£ means each £ can be  
exchanged for HK\$10.



## Exercise 12C

### Level 1

- Express the rates of the following in the units stated in the brackets.
  - A train travels 220 km in 2 hours. (km/h)
  - 3 m<sup>3</sup> of water weighs 3 000 kg. (kg/m<sup>3</sup>)
  - A 4 m<sup>2</sup> carpet costs \$324. (\$/m<sup>2</sup>)
  - 8 pieces of bricks weigh 16.8 kg. (kg/piece)
  - 5 persons live in a house of area 54 m<sup>2</sup>. (m<sup>2</sup>/person)
  - The rent of a flat is \$13 500 for 3 months. (\$/month)
  - 5 pieces of chewing gum cost \$3.8. (\$/piece)
  - The price of  $\frac{1}{2}$  kg of potatoes is \$12.5. (\$/kg)
- Kate can read 60 pages of a book in 40 minutes. Find her reading speed
  - in pages/min.
  - in pages/h.
- A racing car completed a 900 km race in 5 hours. Find the speed of the car
  - in km/h.
  - in m/s.
- 3 cans of 2.9 L peanut oil are sold for \$174. Find the price of peanut oil
  - in \$/can.
  - in \$/L.



5. The area of city A is around  $1\,100\text{ km}^2$  and the population is around  $7\,018\,000$ . The area of city B is around  $700\text{ km}^2$  and the population is around  $4\,354\,000$ .
- (a) Find the population density of city A in persons/ $\text{km}^2$ .
  - (b) Find the population density of city B in persons/ $\text{km}^2$ .
  - (c) Which city is more densely populated?
6. A package of 8 cans of cola is sold for \$20. A package of 24 cans of cola is sold for \$52.8.
- (a) Find the respective prices of the two packages of cola in \$/can.
  - (b) From the result of (a), which package is more economical?
7. In fast-food restaurant A, 5 cooks prepare 290 lunch sets in an hour. In fast-food restaurant B, 8 cooks prepare 480 lunch sets in an hour.
- (a) Find the rate of preparing lunch sets by each cook in each restaurant in an hour.
  - (b) Which fast-food restaurant is more efficient?
8. Flat A with an area of  $85\text{ m}^2$  was sold for \$2.72 million. Flat B with an area of  $75\text{ m}^2$  was sold for \$2.25 million.
- (a) Find the price of each flat in \$/ $\text{m}^2$ .
  - (b) In terms of the price of each  $\text{m}^2$ , which flat was cheaper?
9. John ran 4.5 km in 18 minutes. Peter ran 7 020 m in  $\frac{1}{2}$  hour.
- (a) Find the speeds of John and Peter in m/s.
  - (b) Whose speed was higher?

**Level 2**

10. The hourly salary of a shopkeeper of a store is \$28. He works 8 hours each day.
- (a) Find his salary in \$/day.
  - (b) How much can he earn in 25 days?
11. An aeroplane travelled 5 760 km in 8 hours.
- (a) Find the speed of the aeroplane in km/h.
  - (b) How far can it travel in 4.5 hours?



12. The price of 2 kg of watermelon is \$13.
- Find the price of watermelon in \$/kg.
  - How much watermelon in kg can be bought for \$45.5?

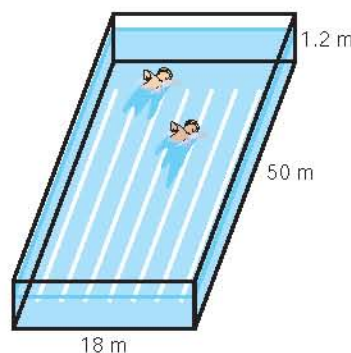


13. A family consumes 5 kg of rice in 20 days.
- Find the rice consumption of this family in kg/day.
  - How much rice is needed for a family in 30 days?
  - If there is 17 kg of rice, how long can it last for in this family?
14. Suppose we can exchange 5 US dollars for 39 HK dollars and 4 Canadian dollars for 27 HK dollars.
- How much HK dollars can we exchange for 100 US dollars?
  - How much HK dollars can we exchange for 100 Canadian dollars?
  - A model of mobile phone is sold for 100 US dollars in the USA and 100 Canadian dollars in Canada. In which country is the mobile phone more expensive?

15. A train can carry 2 400 passengers in total. During rush hours, the train comes every  $1\frac{1}{2}$  minutes.
- Find the frequency of trains in rush hours in runs/h.
  - Find the load in rush hours in passengers/h.
  - How long will it take to carry 144 000 passengers in rush hours?



16. The length, width and depth of a swimming pool are 50 m, 18 m and 1.2 m respectively.
- Find the capacity of the swimming pool.
  - The pool has been empty originally and is now filled up with water at a speed of  $2.4 \text{ m}^3/\text{min}$  by a pipe. How much water is filled in one hour?
  - Suppose after filling the pool for an hour, the speed of water filling the pool increases to  $3 \text{ m}^3/\text{min}$ . Can the swimming pool be filled completely in the next 5 hours? Explain briefly.





## 12.4 Scale Plan

### Class Activity 12.3

**Aim:** To investigate the usage of scale

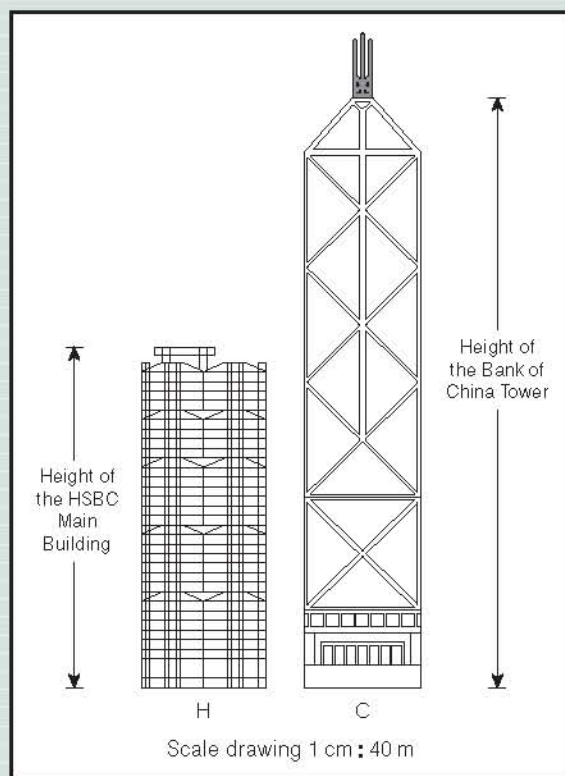
The figure shows a scale drawing of the HSBC Main Building (H) and the Bank of China Tower (C), according to the ratio of their actual heights. It is known that 1 cm in the drawing represents an actual length of 40 m.

- (a) Find the ratio of the height in the drawing to the actual height in the form of  $1 : n$ .

$1 : 4\,000$

- (b) Measure the heights of H and C in the drawing, and calculate their actual heights.

	Height in drawing (cm)	Actual height (m)
H	4.5	180
C	7.8	312



*Now I see ...*

If the ratio of the length of an object in a scale drawing to that of the real object is known, we can find the length of the real object by measuring the length of the object in the drawing.



Have you ever seen a ratio  $1 : 20\,000$  on a map?  $1 : 20\,000$  means that 1 cm on the map represents an actual distance of 20 000 cm. We call this ratio a **scale**.

scale 比例尺

The scale on a map is usually expressed in the form of  $1 : n$ . The following figure shows a map of Hong Kong Island with the scale  $1 : 120\,000$ .



Figure 12.1

$$\begin{aligned}\text{Scale of a map} &= \text{Distance on the map} : \text{Actual distance} \\ &= 1 : n\end{aligned}$$

For example, on a map with scale  $1 : 120\,000$ , the length of 1 cm represents an actual distance of  $1 \times 120\,000 \text{ cm} = 1\,200 \text{ m} = 1.2 \text{ km}$ .

### Example 12.19 Finding a scale

The scale of a map is 2 cm to 5 km. Express the scale in the form of  $1 : n$ .

#### Solution

$$\begin{aligned}5 \text{ km} &= 5 \times 1\,000 \times 100 \text{ cm} & \leftarrow \begin{array}{l} 1 \text{ km} = 1\,000 \text{ m} \\ 1 \text{ m} = 100 \text{ cm} \end{array} \\ &= 500\,000 \text{ cm}\end{aligned}$$

$$\text{The scale} = 2 : 500\,000$$

$$\begin{aligned}&= \frac{2}{500\,000} \\ &= \frac{1}{250\,000} \\ &= \underline{\underline{1 : 250\,000}}\end{aligned}$$

### Classwork 12.19

The following are the scales of maps. Express each of them in the form of  $1 : n$ .

(a) 5 cm to 2 km

(b) 2 cm to  $\frac{1}{5}$  m

**Example 12.20** Application of scale

The scale of a map is 1 : 20 000.

- (a) Find the distance on the map representing 1 km of actual distance.  
 (b) Find the actual length of a river if it is 13.5 cm on the map.

**Solution**

$$(a) \quad \therefore \frac{\text{Distance on the map}}{\text{Actual distance}} = \frac{1}{20\,000}$$

$$\begin{aligned} \therefore \text{Distance on the map} &= \frac{1}{20\,000} \times \text{Actual distance} \\ &= \frac{1}{20\,000} \times 1\text{ km} \\ &= \frac{1}{20\,000} \times 1\,000 \times 100\text{ cm} \\ &= \underline{\underline{5\text{ cm}}} \end{aligned}$$

$$(b) \quad \therefore \frac{\text{Actual distance}}{\text{Distance on the map}} = \frac{20\,000}{1}$$


$$\begin{aligned} \therefore \text{The actual length of the river} &= 20\,000 \times 13.5\text{ cm} \\ &= 270\,000\text{ cm} \\ &= \frac{270\,000}{1\,000 \times 100}\text{ km} \\ &= \underline{\underline{2.7\text{ km}}} \end{aligned}$$

**Classwork 12.20**

The scale of a map is 1 : 5 000.

- (a) Find the distance on the map if the actual distance is 1 km.  
 (b) Find the actual length of a highway if it is 12 cm on the map.

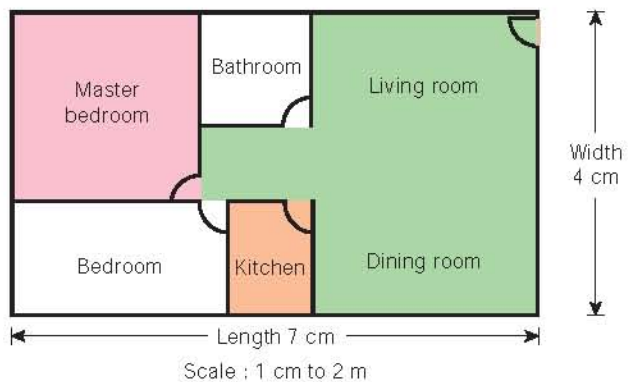
**Skills Upgrading Corner 12.4**

1. (a) The scale of the map is 1 : \_\_\_\_\_.  
 (b) Use the map to find the actual length of Western Harbour Crossing (i.e.  on the map).  
 (c) How long does it take to drive through Western Harbour Crossing at a speed of 60 km/h?





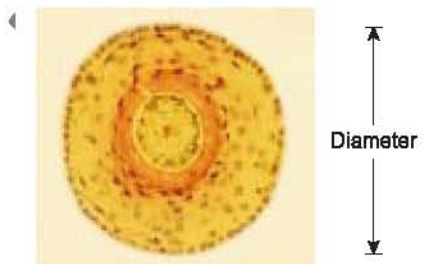
2. The figure shows the floor plan of a flat. Find
- the actual length and width of the flat.
  - the actual area of the flat.



## Exercise 12D

### Level 1

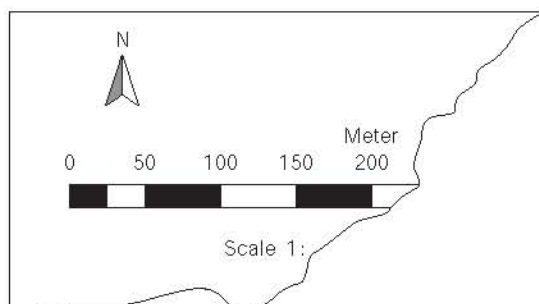
- Express the following map scales in the form of  $1 : n$ .
  - 2 cm to 100 m
  - 4 m to 1 km
  - 5 cm to  $\frac{1}{2}$  km
- On the floor plan of a playground, a 100 m-track is 20 cm long. Find the scale in the form of  $1 \text{ cm} : n \text{ m}$ .
- The figure shows a picture of the cross-section of human hair.
  - Measure the diameter of the hair in the picture.
  - If the actual diameter of a hair is 0.12 mm, find the scale of the figure in the form of  $1 : n$ .
- It is given that the scale of a map is 2 cm to 50 km. If the actual distance between two cities is 210 km, what is the distance between the cities on the map?
- The scale of a map is 1 cm : 1 km. If two railway stations are 2.8 cm apart on the map, find the actual distance between the stations.
- The figure is a map showing the West Rail Line of MTR. If the scale of the map is  $1 : 500\,000$  and the length of the railway on the map is 6.1 cm, find the actual length of the West Rail Line.



7. The figure is a drawing of an *ornament*. A pearl will be placed in the circular region at the centre. If the diameter of the circular region at the centre is 32 mm in the drawing, and the scale of the drawing is 1 : 0.25, find the actual diameter of the circular region.



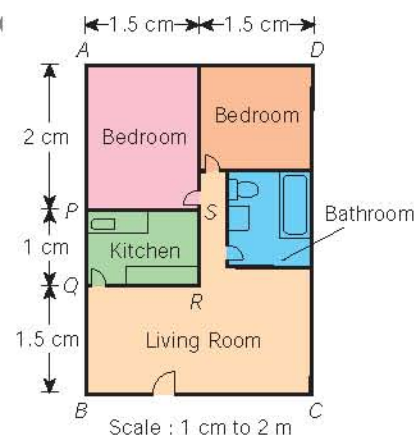
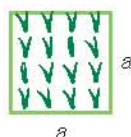
8. The figure is a corner of a map.



- (a) Find the scale of the map in the form of 1 :  $n$ .  
 (b) Find the distance between two washrooms on the map which are in fact 750 m apart.
9. The scale of a map is 1 : 200 000.
- (a) Find the scale in the form of 1 cm :  $n$  km.  
 (b) (i) Find the distance on the map representing the actual distance 125 km.  
 (ii) Find the actual distance represented by 16.5 cm on the map.

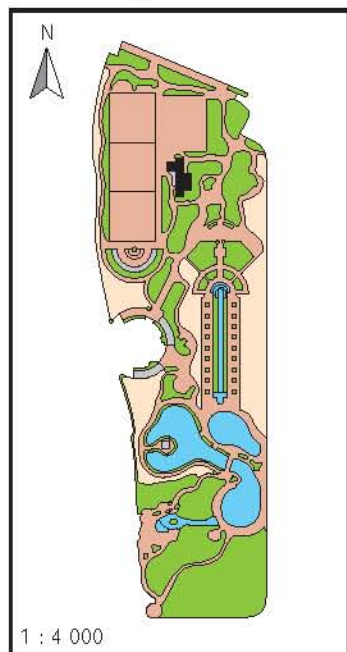
## Level 2

10. On a floor plan with a scale of 1 : 100, the length and width of a classroom are 5 cm and 6 cm respectively.
- (a) Find the actual dimensions of the classroom. (Give your answer in Length  $\times$  Width.)  
 (b) Find the actual area of the classroom.
11. The figure shows the floor plan of a flat with a scale of 1 cm to 2 m.
- (a) Find the actual dimensions of the kitchen. (Give your answer in Length  $\times$  Width.)  
 (b) Find the actual area of the kitchen.
12. On a map with a scale of 1 : 1 000, the sides of a square farm are  $a$  each.

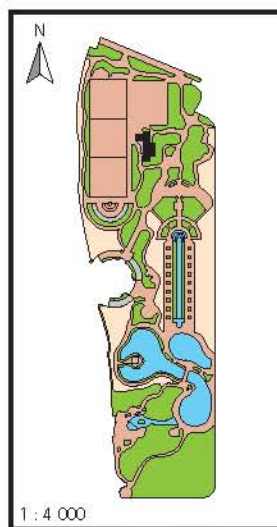


- (a) Find the actual length of the farm in terms of  $a$ .  
 (b) Find the actual area of the farm in terms of  $a$ .  
 (c) Find the ratio of the area of the farm on the map to its actual area.

13. The scale of a floor plan I is 1 : 4 000. It is reduced in size to a floor plan II such that the length of floor plan II is  $\frac{4}{5}$  of that of floor plan I.



Floor plan I



Floor plan II

- (a) Explain why the scale 1 : 4 000 shown in floor plan II is incorrect.
- (b) (i) Find the length on floor plan I representing an actual distance of 1 000 m.  
 (ii) Find the length on floor plan II representing an actual distance of 1 000 m.
- (c) Find the scale of floor plan II in the form of 1 :  $n$ .



## Chapter Summary

### A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

#### Column A

1. Ratio •
2. Rate •

#### Column B

- (a) The quotient of two quantities.
- (b) The quotient of two quantities of the same kind.

### B. Fact to Remember

1. The ratio of  $a$  and  $b$  is expressed as  $\frac{a}{b}$  or  $a : b$ , where  $a \neq 0$  and  $b \neq 0$ .
2. The scale of a map = Distance on the map : Actual distance





## Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

1. (a) There are 26 passengers on the upper deck and 39 passengers on the lower deck of a bus. The ratio of the number of passengers on the upper deck to that on the lower deck is \_\_\_\_\_.  
(b) The ratio of Tony's weight to Karen's is 7 : 4. If Karen weighs 36 kg, find the weight of Tony.
2. (a) If  $x : 24 : 12 = 5 : 2 : 1$ ,  $x =$  \_\_\_\_\_.  
(b) It is given that the white, red and blue paints are mixed in the ratio of 3 : 4 : 2. If the volume of blue paint is 400 mL, find the total volume of the mixed paint.
3. (a) A 6-pack lemon tea is sold for \$12.6.  
The selling price of lemon tea = \$ \_\_\_\_\_.  
(b) The typing speed of Fanny is 80 words/min. How many words can she type in 30 minutes?
4. It is known that the scale of a map is 1 : 50 000.  
(a) The actual distance between two points represented by 1.5 cm on the map is \_\_\_\_\_ km.  
(b) What is the distance between two points on the map representing the actual distance of 1.5 km?

Section

12.1

12.2

12.3

12.4



## Revision Exercise 12

### Level 1

1. Express the following rates in the units stated in the brackets.
  - (a) A piece of cloth selling at \$96 for 3 m. (\$/m)
  - (b) Eggs selling at \$4.8 per dozen. (\$/egg)
  - (c) 54 candies weighing 0.5 kg. (candies/kg)
  - (d) 5 tins of petrol weighing 80 kg. (kg/tin)

2. Simplify the following ratios.

(a)  $39 : 630$

(b)  $280 : 560$

(c)  $\frac{3}{2} : \frac{9}{4}$

(d)  $\frac{7}{5} : \frac{3}{15}$

(e)  $0.125 : 0.375$

(f)  $2.4 : 6.4$

3. Simplify the following ratios.

(a) 2 days : 32 hours

(b) 56 cm : 4 m

(c) \$4 : 25¢

(d) 2 minutes : 90 seconds

(e) 3.6 kg : 800 g

(f)  $\frac{3}{5}$  of a right angle :  $270^\circ$

4. Find the values of the unknowns in the following.

(a)  $7 : 2 = 98 : p$

(b)  $5 : 6 = q : 42$

(c)  $32 : 28 = x : 35$

(d)  $\frac{1}{2} : 7 = 5 : a = 2b : 56$

5. It is given that the length of 5 mm in a drawing of a furniture represents the actual length of 60 cm. Find the scale of the drawing in the form of  $1 : n$ .

6. A city has a population of 5 000 000 including 87 500 newborn babies. What is the birth rate for every 1 000 people?

7. The water temperature in an electric boiler raises from  $25^\circ\text{C}$  to  $100^\circ\text{C}$  in 15 minutes. What is the rate of temperature rise in  $^\circ\text{C}/\text{min}$ ?

8. The cost of tiles for covering a  $16\text{ m} \times 12.5\text{ m}$  rectangular room completely is \$38 000. Find the cost of tiles in  $\$/\text{m}^2$ .

9. A horse runs 1 600 m in 1 minute 40 seconds, and a car travels 90 km in  $2\frac{1}{2}$  hours.

(a) Express the speeds of the horse and the car in m/s.

(b) Which is faster?



10. In shop A, eggs are sold for 50¢ each. In shop B, eggs are sold for \$4.8 per dozen. Find the ratio of the price of eggs in shop A to that in shop B.



11. On a map, the distance between two schools is 4 cm. If the scale of the map is 1 cm to 750 m, find the actual distance between these two schools.

12. It is given that the distance between the entrances of two tunnels is 800 m. On a map with the scale  $1 : 20\,000$ , what is the distance between the entrances?

13. Find  $a:b:c$  if
- (a)  $a:b=2:3$ ,  $b:c=2:5$ . (b)  $a:b=4:5$ ,  $a:c=6:7$ .
14. In the following,  $a$ ,  $b$  and  $c$  are non-zero. Find  $a:b:c$ .
- (a)  $a=\frac{1}{2}b$ ,  $b=\frac{3}{4}c$  (b)  $3a=5b$ ,  $3b=2c$
15. If  $a:b:c=3:4:6$ , find the ratios of the following.
- (a)  $\frac{1}{a}:\frac{1}{b}$  (b)  $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$
16. The ratio of the weights of tin and zinc in a 112 kg alloy is 5:2. Find the weight of tin in the alloy.
17. A lamp post is painted in black and white in the ratio of 3:2. If the length of the white portion is 4 m, find the length of the lamp post.

**Level 2**

18. Carmen spends 15%, 30% and 35% of her pocket money on transportation, lunch and savings respectively, and the rest is spent on entertainment. Find the ratio of the amounts spent on transportation, lunch and entertainment.
19. A butcher sells beef at \$20 per catty. A supermarket sells beef of the same quality at \$16.5 per pound. Given that 3 catties equal 4 pounds, beef from which source is cheaper?
20. 480 pieces of chocolate are shared by Frankie and Amy in the ratio of  $7:a$  (where  $a$  is a positive integer). Frankie's share is more than Amy's. Write down one set of possible number of pieces of chocolate got by them.
21. The length of a wire is 360 cm.
- (a) If it is divided into 3 portions in the ratio of 1:3:8, find the length of the longest portion.
- (b) If the wire with the longest portion is bent into a rectangle with its length and width in the ratio of 3:2, find the length and width of the rectangle.
22. Given that  $(a+2):(a+5):b=3:4:7$ , find the values of  $a$  and  $b$ .
23. Lawrence's test scores in Chinese Language, English Language and Mathematics are in the ratio of 6:9:8. If the total score of these three tests is between 220 and 300, write down a set of possible scores in the three tests.





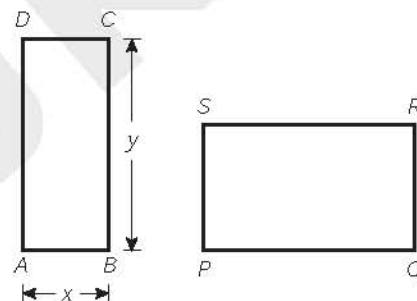
24. The total number of candies got by Leo, Christine and Mandy is 480. Christine gets twice as many as Leo does, while Mandy gets half as many as Christine does.
- Find the ratio of the number of candies got by Leo, Christine and Mandy.
  - Find the number of candies got by each of them.

25. A wheel runs a distance of 27 m in 15 complete turns. Find the number of complete turns required for running 405 m.

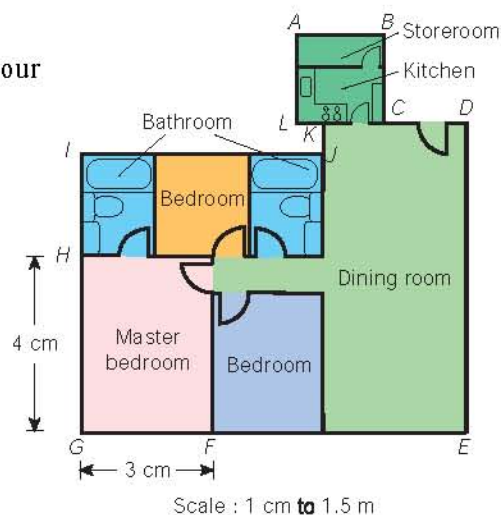


26. Joe took 10 minutes to travel 6 000 m by bicycle. If his speed in the last  $\frac{1}{3}$  of his trip was 500 m/min,
- how long did he take to complete the last  $\frac{1}{3}$  of his trip?
  - what was his speed for the rest of his trip?

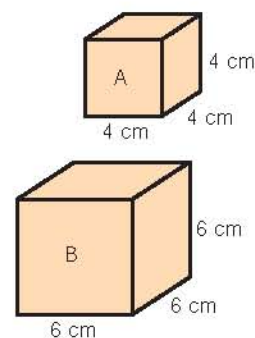
27. The figure shows rectangles  $ABCD$  and  $PQRS$ , where  $AB : PQ = 2 : 5$  and  $BC : QR = 5 : 3$ .
- Express  $PQ$  in terms of  $x$ .
    - Express  $QR$  in terms of  $y$ .
    - Hence, find the ratio of the area of  $ABCD$  to that of  $PQRS$ .
  - If the area of  $ABCD$  is  $70 \text{ cm}^2$ , find the area of  $PQRS$ .



28. The figure shows a floor plan with a scale of 1 cm : 1.5 m.
- Find the actual dimensions of the master bedroom. (Give your answer in Length  $\times$  Width.)
  - Find the actual area of the master bedroom.



29. The sides of cube A and cube B are 4 cm and 6 cm each respectively.
- Find the ratio of the sides of the cubes.
    - Find the ratio of the volumes of the cubes.
  - According to the results of (a)(i) and (a)(ii), what is the relation between the ratio of the lengths of two cubes and the ratio of the volumes of two cubes?



30. Johnny must finish typing a letter of 2 000 words in one hour. In the first 30 minutes, his typing speed was 30 words/min. In the next 15 minutes, his typing speed became 40 words/min, and then he took a break for 5 minutes.
- (a) How many words did he type before the break?
- (b) After the break, he kept on typing at a speed of 40 words/min. Could he finish the letter within the time limit? If not, suggest him a suitable typing speed.
31. There are 3 groups of children. If a bag of candies is shared by children in group A, each child can get 27 candies. Similarly, if the same bag of candies is shared by children in group B and by children in group C, each child can get 36 and 54 candies respectively.
- (a) Let  $x$ ,  $y$  and  $z$  be the number of children in groups A, B and C respectively.
- (i) Express the number of candies in the bag in terms of  $x$ .
- (ii) Express the number of candies in the bag in terms of  $y$ .
- (iii) Express the number of candies in the bag in terms of  $z$ .
- (iv) Find  $x : y : z$ .
- (b) If the candies cost \$36 in total and they are shared equally among all children, what is the cost of group A's share of candies?

**MC Question**

32. 1 km : 20 cm =
- A. 5 000 : 1.  
B. 500 : 1.  
C. 50 : 1.  
D. 1 : 20. ☐
33. If  $12 : 18 = x : 12$ , then  $x =$
- A. 6.  
B. 8.  
C. 12.  
D. 18. ☐
34. There are 12 floors in a shopping centre with 2 male washrooms and 2 female washrooms on each floor. An extra female washroom is going to be added to every even numbered floor. What will be the ratio of the number of male washrooms to that of female washrooms in the shopping centre?
- A. 1 : 1  
B. 4 : 5  
C. 2 : 3  
D. 1 : 2 ☐
35. The length of a bridge is 9.6 cm on a map with the scale 1 : 15 000. The actual length of the bridge is
- A. 0.64 km.  
B. 0.96 km.  
C. 1.44 km.  
D. 1.5 km. ☐
36. Joey typed 6 480 words in 1 hour. Joey's typing speed =
- A. 1.8 words/min.  
B. 64.8 words/min.  
C. 108 words/min.  
D. 6 480 words/min. ☐
37. If  $a : b = 3 : 5$  and  $c : b = 5 : 2$ , then  $a : b : c =$
- A. 2 : 5 : 3.  
B. 3 : 5 : 2.  
C. 6 : 10 : 25.  
D. 15 : 6 : 10. ☐





## Problem-solving and Exploring



### Hint for the Title Page Question

The prices of Happy Cow milk in 125 mL, 250 mL and 1 L are \$2.5, \$4.5 and \$15 respectively.

- Find the price of each package in \$/mL.
- Which package is the most economical? Does it necessarily fit your need?



### Additional Question

- Photo paper in different sizes, e.g. 3R, 4R, etc. are available for photo printing in photofinishing shops. The following are sizes which are commonly used.

Photo	3R	4R	5R	6R	8R	12R
Size	$3.5'' \times 5''$	$4'' \times 6''$	$5'' \times 7''$	$6'' \times 8''$	$8'' \times 10''$	$12'' \times 15''$

(" is a symbol representing the unit of inch,  $1'' = 2.54$  cm.)

- As indicated in the figure, find the ratio  $a : b$  for each size of photo paper.
- Do you know the size of a film used in a traditional camera? Which size of the photo paper above has the same dimension ratio as the dimension ratio of a film?
- Photos taken by most of the digital cameras are in the dimension ratio of  $3 : 4$ . When a 4R-sized photo is printed, there is either a white margin around the photo, or part of the image is chopped away. Can you explain the reason?



- Search the web for the champion records for the following events in the Olympic Games 2004.
    - male 100 m
    - male 400 m
    - male 10 000 m
    - female 100 m
    - female 400 m
    - female 10 000 m
  - Find the speeds (in m/s) of the champions in (a).
  - From (b), between the male 100 m and male 400 m champions, which one has a higher speed? Between the male 400 m and male 10 000 m champions, which one has a higher speed?







# Appreciating Mathematics

## Enrichment

### Golden Ratio

#### A Introduction to golden ratio

According to the following construction, a rectangle  $ABEF$  can be constructed.

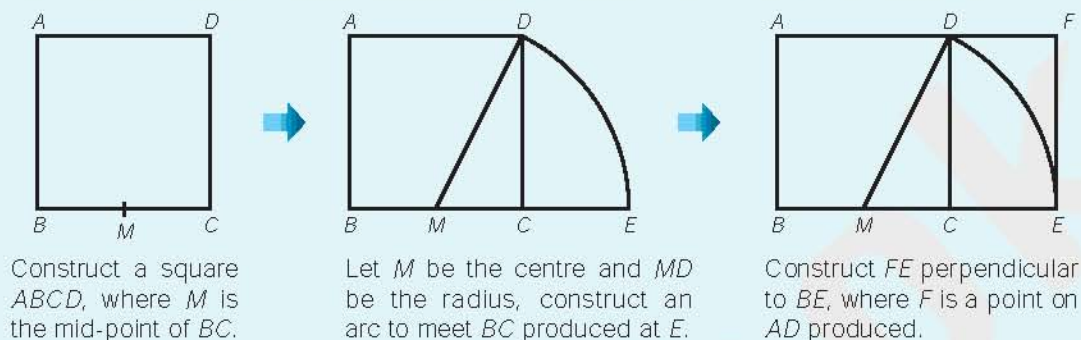


Figure 12.2

Rectangle  $ABEF$  obtained from the above construction is called golden rectangle, and the ratio between  $AB$  and  $BE$  is called golden ratio. The value of the golden ratio is about 0.618.

i.e.  $AB : BE \approx 0.618 : 1$ .

#### B Fibonacci sequence and golden ratio

The famous Fibonacci sequence is closely related to the golden ratio. Consider the first 12 terms in the sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

From the ratios of each term in the sequence to each of its following term correct to 3 decimal places, we discover that the ratio between two consecutive terms of Fibonacci sequence will gradually approach to the golden ratio 0.618. (See Figure 12.3)

According to this relation, we can draw an approximate golden rectangle with squares. The rectangle below is drawn by using the first 8 terms of the Fibonacci sequence. The ratio between the length and width is  $21 : 34 \approx 0.618 : 1$ .

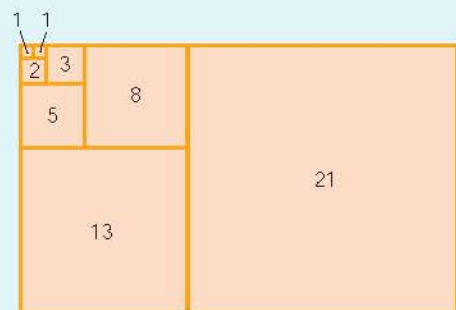


Figure 12.4

$\frac{1}{1} = 1$
$\frac{1}{2} = 0.5$
$\frac{2}{3} = 0.667$
$\frac{3}{5} = 0.6$
$\frac{5}{8} = 0.625$
$\frac{8}{13} = 0.615$
$\frac{13}{21} = 0.619$
$\frac{21}{34} = 0.618$
$\frac{34}{55} = 0.618$
$\frac{55}{89} = 0.618$
$\frac{89}{144} = 0.618$

Figure 12.3

## C Golden ratio and art

Many pieces of art related to the golden ratio have been created since the ancient world, and most of them are publicly recognized works of art.

The Parthenon has fully applied golden rectangle on its design, which brings people the feeling of *harmony* and *solemnity*. (See Figure 12.5)

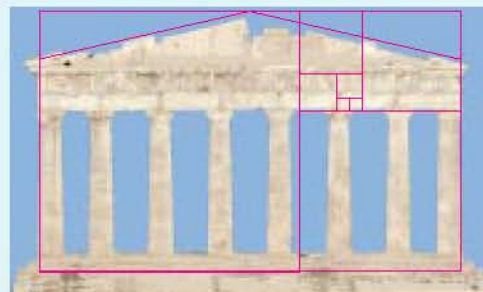
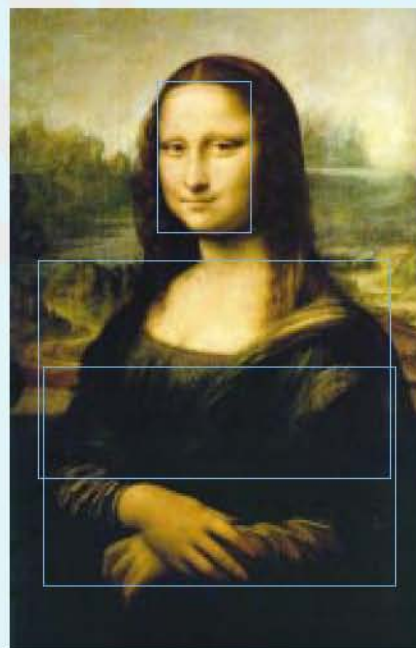


Figure 12.5

Leonardo da Vinci, an outstanding Italian inventor in the 16th century and also a great artist of the time, has actually applied golden ratio in his *masterpiece* Mona Lisa. With the background drawn by *perspective approach*, the lady in the picture shows natural beauty. (See Figure 12.6)



The rectangles in the figure are golden rectangles.

Figure 12.6

The Pythagorean School of ancient Greece and the national flag of the People's Republic of China have used *pentagram* on the design. (See Figure 12.7)

In fact, the sides of the pentagram used have also applied the golden ratio, for example, in the figure,

$$BC : CD = AC : AD \approx 0.618 : 1$$

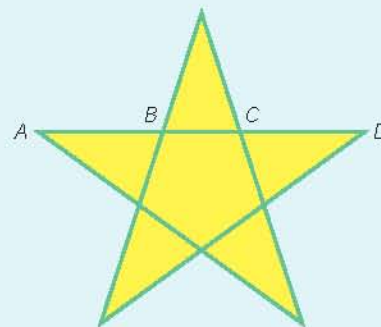


Figure 12.7



On our national flag, the position of the large pentagram is placed approximately according to golden ratio.



Figure 12.8

In the figure,  $a : b \approx 0.6 : 1$

### **D** Golden ratio in nature

Golden ratio seems to be applied in our nature as well.

Take the example of *nautilus* found in South China Sea. The pattern of the spiral curves on the shell of a nautilus can also be obtained by golden rectangle. (See Figure 12.9)

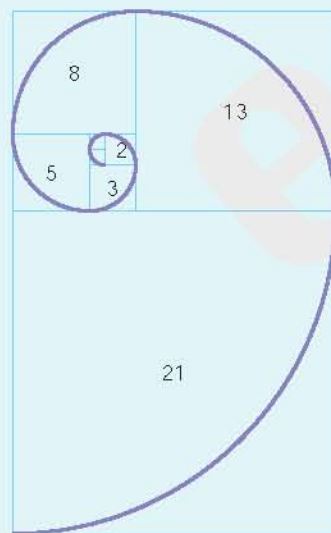


Figure 12.9



Figure 12.10

This kind of spiral curves is called equiangular spiral, and this can be found on the shells of many marine creatures and snails.



Moreover, the golden ratio seems to be hidden in different parts of our body too! (See Figure 12.11)

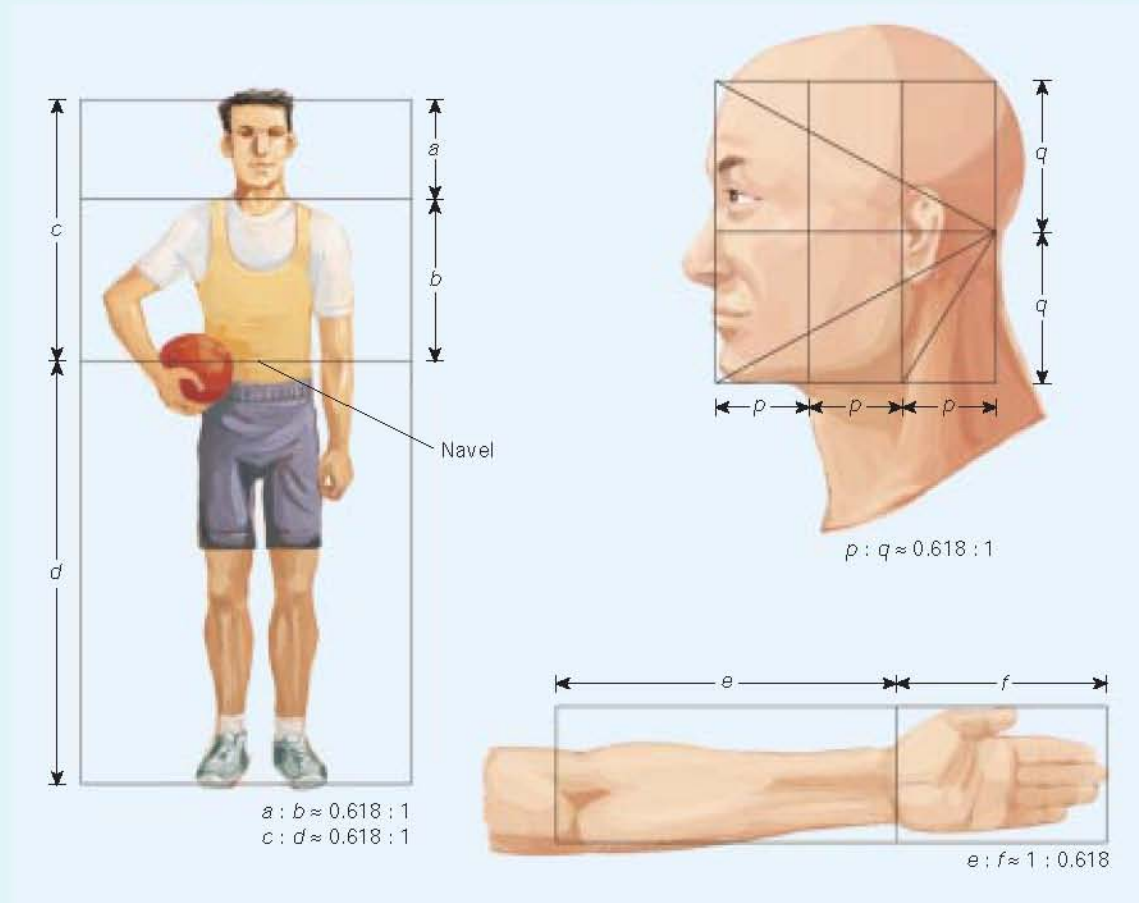


Figure 12.11

Perhaps this is why things that applied golden ratio give a sense of beauty.